



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

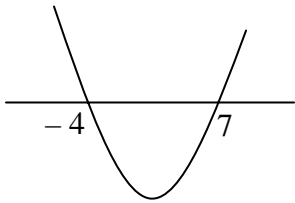
GRADE 12

**MATHEMATICS P1
ADDITIONAL EXEMPLAR 2008
MEMORANDUM**

MARKS: 150

This memorandum consists of 11 pages.

QUESTION 1

<p>1.1.1</p>	$\frac{1}{x} + \frac{5}{x-1} = 6$ $x-1+5x = 6x(x-1)$ $x-1+5x = 6x^2 - 6x$ $0 = 6x^2 - 12x + 1$ $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(6)(1)}}{2(6)}$ $x = \frac{12 \pm \sqrt{120}}{12}$ $x = 1,91 \text{ or } x = 0,09$	<ul style="list-style-type: none"> ✓ simplification ✓ standard form ✓ substitution ✓ simplification ✓✓ answers (6)
<p>1.1.2</p>	$x^2 - 3x \geq 28$ $x^2 - 3x - 28 \geq 0$ $(x-7)(x+4) \geq 0$ $\begin{array}{c} + \quad 0 \quad - \quad 0 \quad + \\ \hline -4 \quad 7 \end{array} \rightarrow \text{OR}$  $x \leq -4 \text{ or } x \geq 7$ OR $x \in (-\infty; -4] \cup [7; \infty)$	<ul style="list-style-type: none"> ✓ standard form ✓ factors ✓ method ✓✓ answer (5)
<p>1.2</p>	$2x - y = 3$ $y = 2x - 3$ $x^2 + 5x(2x - 3) + (2x - 3)^2 = 15$ $x^2 + 10x^2 - 15x + 4x^2 - 12x + 9 - 15 = 0$ $15x^2 - 27x - 6 = 0$ $5x^2 - 9x - 2 = 0$ $(5x+1)(x-2) = 0$ $x = -\frac{1}{5} \text{ or } x = 2$ $y = -\frac{17}{5} \text{ or } y = 1$	<ul style="list-style-type: none"> ✓ simplification ✓ substitution ✓ simplification ✓ standard form ✓ factors ✓ answers ✓ answers

OR	
$2x - y = 3$ $x = \frac{y+3}{2}$ $\left(\frac{y+3}{2}\right)^2 + 5\left(\frac{y+3}{2}\right)y + y^2 = 15$ $\frac{y^2 + 6y + 9}{4} + \frac{5y^2 + 15y}{2} + y^2 = 15$ $y^2 + 6y + 9 + 10y^2 + 30y + 4y^2 - 60 = 0$ $15y^2 + 36y - 51 = 0$ $5y^2 + 12y - 17 = 0$ $(5y+17)(y-1) = 0$ $y = -\frac{17}{5} \text{ or } y = 1$ $x = -\frac{1}{5} \text{ or } x = 2$	(7) [18]

QUESTION 2

2.1	$T_n = -5 + (n-1)(4)$ $439 = -5 + 4(n-1)$ $444 = 4(n-1)$ $n-1 = 111$ $n = 112$ <p style="text-align: center;">OR</p> $T_n = 4n - 9$ $439 = 4n - 9$ $448 = 4n$ $112 = n$	$\checkmark T_n = 4n - 9$ $\checkmark \text{ substitution of } 439$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>
2.2.1	$r = \frac{27p^2}{81p} = \frac{p}{3}$	$\checkmark \text{ ratio}$ $\checkmark \text{ simplification}$ <p style="text-align: right;">(2)</p>
2.2.2	$-1 < \frac{p}{3} < 1$ $-3 < p < 3 \quad p \neq 0$	$\checkmark \text{ setting up inequality}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>

<p>2.2.3</p>	<p>If $p = 2$ the sequence is 162 ; 108 ; 72 ; 48 ;</p> $\therefore a = 162 \quad ; \quad r = \frac{2}{3}$ $S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{162}{1-\frac{2}{3}}$ $S_{\infty} = 486$	<p>✓ value of a and r</p> <p>✓ formula</p> <p>✓ substitution</p> <p>✓ answer</p> <p style="text-align: right;">(3) [11]</p>
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QUESTION 3

<p>3.1</p>	<p>Tebogo's sequence will form a geometric sequence with common ratio 3. Thembe's sequence will form a quadratic sequence with a constant second difference 8.</p>	<p>✓✓ explanation (Tebogo)</p> <p>✓✓ explanation (Thembe)</p> <p style="text-align: right;">(4)</p>
<p>3.2</p>	$2 \quad \diagdown \quad 4 \quad \diagup \quad 6 \quad \diagdown \quad 12 \quad \diagup \quad 18 \quad \diagdown \quad 20 \quad \diagup \quad 38$ $2a = 8$ $a = 4$ $T_n = 4n^2 + bn + c$ $2 = 4 + b + c$ $-2 = b + c$ $6 = 4(2)^2 + 2b + c$ $-10 = 2b + c$ $b = -8$ $c = 6$ $T_n = 4n^2 - 8n + 6 \text{ (Thembe's sequence)}$ <p style="text-align: center;">OR</p> $2a = 8$ $a = 4$ $T_0 = 6 = c$ $T_1 = 2 = 4 + b + 6$ $b = -8$ $T_n = 4n^2 - 8n + 6 \text{ (Thembe's sequence)}$ <p>Tebogo's sequence is</p> $T_n = 2 \cdot 3^{n-1}$	<p>✓ $a = 4$</p> <p>✓✓ setting up equations</p> <p>✓ value of b</p> <p>✓ value of c</p> <p>✓✓ answer</p> <p style="text-align: right;">(7)</p>
<p>3.3</p>	$T_n = 4n^2 - 8n + 6$ $T_{11} = 4(11)^2 - 8(11) + 6$ $T_{11} = 402$	<p>✓ substitution</p> <p>✓ answer</p> <p style="text-align: right;">(2)</p>

3.4	$S_n = \frac{a(r^n - 1)}{r - 1}$ $531440 = \frac{2(3^n - 1)}{3 - 1}$ $531440 = 3^n - 1$ $531441 = 3^n$ $3^{12} = 3^n$ $n = 12$	✓ substitution ✓ simplification ✓ answer (3) [16]
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QUESTION 4

4.1	$f(x) = -(x+1)^2 + 4$ $0 = -(x+1)^2 + 4$ $(x+1)^2 = 4$ $x+1 = 2 \quad \text{or} \quad x+1 = -2$ $x = 1 \quad \text{or} \quad x = -3$ <p style="text-align: center;">OR</p> $f(x) = -(x+1)^2 + 4$ $0 = -x^2 - 2x - 1 + 4$ $x^2 + 2x - 3 = 0$ $(x-1)(x+3) = 0$ $x = 1 \quad \text{or} \quad x = -3$ $A(-3 ; 0) \text{ and } B(1 ; 0)$	✓ $f(x) = 0$ ✓ factors ✓✓ answer (4)
4.2	$C(0 ; 3)$ $y = a \cdot 3^x + 3$ $4 = a \cdot 3^{-1} + 3$ $1 = \frac{a}{3}$ $a = 3$ $g(x) = 3 \cdot 3^x + 3$	✓ $q = 3$ ✓ substitution ✓ answer (3)
4.3	$f(x) = -x^2 - 2x + 3$ $f'(x) = -2x - 2$ $1 = -2x - 2$ $3 = -2x$ $x = -\frac{3}{2}$ $\text{Point} \left(-\frac{3}{2} ; \frac{15}{4} \right)$	✓ $f'(x) = -2x - 2$ ✓ equating to 1 ✓ answer for x ✓ answer for y (4)
4.4	$k > 4$	✓✓ answer (2) [13]

QUESTION 5

5.1	$p(x) = a^x$ $8 = a^{-3}$ $8 = \frac{1}{a^3}$ $a^3 = \frac{1}{8}$ $a = \frac{1}{2}$	✓ substitution ✓ simplification ✓ answer (3)
5.2	$y = \log_{\frac{1}{2}} x$ OR $y = -\log_2 x$ OR $y = \log_2 \frac{1}{x}$	✓ answer (2)
5.3	$0 < x < 8$	✓✓ answer (2)
5.4	$q(x) = \left(\frac{1}{2}\right)^{x-3}$ OR $q(x) = 2^{-x+3}$	✓✓ answer (2) [9]

QUESTION 6

6.1	$y \in R - \{5\}$ OR $y \in (-\infty; 5) \cup (5; \infty)$	✓✓ answer (2)
6.2	$\frac{-3}{x+1} + 5 = -3x + 2$ $-3 + 5(x+1) = -3x(x+1) + 2(x+1)$ $-3 + 5x + 5 = -3x^2 - 3x + 2x + 2$ $3x^2 + 6x = 0$ $3x(x+2) = 0$ $x = 0$ or $x = -2$ Points of intersection are (0 ; 2) and (-2 ; 8)	✓ equating ✓ simplification ✓✓ answers (4)
6.3	Reflection about the asymptote $x = -1$ OR Reflection about the asymptote $y = 5$ OR Reflection about the x -axis and translated up by 10 units	✓✓ answer (2) [8]

QUESTION 7

7.1	180^0	✓ answer (1)
7.2	$x = -45^0$	✓✓ answer (2)
7.3	$k(x) = -\sin 2x$	✓✓ answer (2)
		[5]

QUESTION 8

8.	<p>let $n = 2008$</p> $2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$ $= n^2 + (n+1)(n-1) - (n-2)(n+2) - (n+8)(n-8)$ $= n^2 + n^2 - 1 - n^2 + 4 - n^2 + 64$ $= 67$ <p style="text-align: center;">OR</p> <p>Let $n = 2000$</p> $2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$ $= (n+8)^2 + (n+9)(n+7) - (n+6)(n+10) - n(n+16)$ $= n^2 + 16n + 64 + n^2 + 16n + 63 - (n^2 + 16n + 60) - (n^2 + 16n)$ $= 64 + 63 - 60$ $= 67$	<ul style="list-style-type: none"> ✓ let $n = 2008$ ✓ substitution ✓ simplification ✓ answer <p style="text-align: right;">[4]</p>
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QUESTION 9

9.1	$35000 = 5000 \left(1 + \frac{0,096}{4} \right)^{4n}$ $7 = \left(1 + \frac{0,096}{4} \right)^{4n}$ $\log 7 = \log \left(1 + \frac{0,096}{4} \right)^{4n}$ $\log 7 = 4n \log \left(1 + \frac{0,096}{4} \right)$ $4n = 82,0486988\dots$ $n = 20,51 \text{ years}$	<ul style="list-style-type: none"> ✓ $4n$ ✓ substitution ✓ log both sides ✓ use of power law ✓ answer <p style="text-align: right;">(5)</p>
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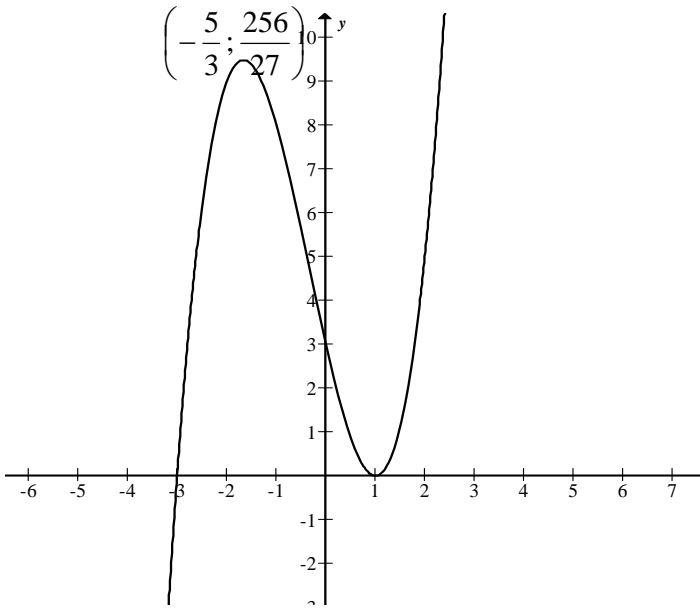
9.2.1	$192000 = \frac{x \left[1 - \left(1 + \frac{0,12}{12} \right)^{-60} \right]}{\frac{0,12}{12}}$ $1920 = x \left[1 - \left(1 + \frac{0,12}{12} \right)^{-60} \right]$ $x = R\ 4\ 270,93$	<ul style="list-style-type: none"> ✓ $\frac{0,12}{12}$ ✓ substitution ✓ 60 months ✓ answer <p style="text-align: right;">(4)</p>
9.2.2	<p>Balance Outstanding</p> $= 192000 \left(1 + \frac{0,12}{12} \right)^{45} - \frac{4270,93 \left[\left(1 + \frac{0,12}{12} \right)^{45} - 1 \right]}{\frac{0,12}{12}}$ $= 300\ 443,66 - 241\ 226,7165\dots$ $= R\ 59\ 216,95$	<ul style="list-style-type: none"> ✓ $\frac{0,12}{12}$ ✓ 45 ✓ 4270,93 ✓✓ substitution ✓✓ answer <p style="text-align: right;">(7) [16]</p>

QUESTION 10

10.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$ $= 3x^2$	<ul style="list-style-type: none"> ✓ substitution ✓ simplification ✓ simplification ✓ common factor ✓ answer <p style="text-align: right;">(5)</p>
10.2.1	$y = \frac{2}{5\sqrt{x}} - \sqrt[3]{x}$ $y = \frac{2}{5}x^{-\frac{1}{2}} - x^{\frac{1}{3}}$ $\frac{dy}{dx} = -\frac{1}{5}x^{-\frac{3}{2}} - \frac{1}{3}x^{-\frac{2}{3}}$ $= -\frac{1}{5\sqrt{x^3}} - \frac{1}{3\sqrt[3]{x^2}}$	<ul style="list-style-type: none"> ✓✓ simplification (one per term) ✓✓ answer <p style="text-align: right;">(4)</p>

<p>10.2.2</p>	$y = \frac{x^4 - 3x^2 + 7}{x}$ $y = x^3 - 3x + 7x^{-1}$ $\frac{dy}{dx} = 3x^2 - 3 - 7x^{-2}$ $= 3x^2 - 3 - \frac{7}{x^2}$	<p>✓✓ simplification</p> <p>✓✓ answer</p> <p style="text-align: right;">(4) [13]</p>
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QUESTION 11

<p>11.1</p>	$f(x) = x^3 + x^2 - 5x + 3$ $0 = (x - 1)(x^2 + 2x - 3)$ $0 = (x - 1)(x - 1)(x + 3)$ $x = 1 \text{ or } x = -3$ <p>x-intercepts are (1 ; 0) and (-3 ; 0)</p> <p>y-intercept is (0 ; 3)</p>	<p>✓ $f(x) = 0$</p> <p>✓ $(x - 1)$</p> <p>✓ $(x^2 + 2x - 3)$</p> <p>✓ x-intercepts</p> <p>✓ y-intercept</p> <p style="text-align: right;">(5)</p>
<p>11.2</p>	$f'(x) = 3x^2 + 2x - 5$ $0 = 3x^2 + 2x - 5$ $0 = (3x + 5)(x - 1)$ <p>Turning Points are</p> $(1 ; 0) \text{ and } \left(-\frac{5}{3}; \frac{256}{27}\right) \text{ or } \left(-\frac{5}{3}; 9,48\right)$	<p>✓ $f'(x) = 3x^2 + 2x - 5$</p> <p>✓ $f'(x) = 0$</p> <p>✓ factors</p> <p>✓ answer</p> <p>✓ answer</p> <p style="text-align: right;">(5)</p>
<p>11.3</p>	$f''(x) = 6x + 2$ $0 = 6x + 2$ $x = -\frac{1}{3}$	<p>✓ $f''(x) = 6x + 2$</p> <p>✓ $f''(x) = 0$</p> <p>✓ answer</p> <p style="text-align: right;">(3)</p>
<p>11.4</p>		<p>✓ intercepts</p> <p>✓ turning points</p> <p>✓ shape</p> <p style="text-align: right;">(3) [16]</p>

QUESTION 12

12.1	Height of the cylinder = $2x$ By Pythagoras $(\text{radius of cylinder})^2 = (5\sqrt{3})^2 - x^2$ $= 75 - x^2$ $V = \pi r^2 h$ $V = \pi(75 - x^2)2x$ $V = 150\pi x - 2\pi x^3$	✓ substitution ✓ answer for r ✓ substitution (3)
12.2	$\frac{dV}{dx} = 150\pi - 6\pi x^2$ $0 = 150\pi - 6\pi x^2$ $x^2 = 25$ $x = \pm 5$ $x = 5 \quad x > 0$ Height of the cylinder = $2(5)$ cm $= 10$ cm	✓ $\frac{dV}{dx} = 150\pi - 6\pi x^2$ ✓ $\frac{dV}{dx} = 0$ ✓ answer for x ✓ answer (4) [7]

QUESTION 13

13.1	$x + y \leq 16$ $120x + 60y \geq 1200$ $2000x + 3000y \geq 36000$ $x, y \in N_0$	✓ answer ✓ answer ✓ answer ✓ answer (4)
13.2		✓✓✓ graphs of constraints ✓ feasible region (4)
13.3	$C = 40000x + 48000y$	✓ answer (1)
13.4	$m = -\frac{40000}{48000}$ $m = -\frac{5}{6}$ Minimum Cost at (6 ; 8) i.e. 6 Silver Jets and 8 Golden Flyers	✓ gradient of search line ✓✓ answer (3)
13.5	$C = 40000(6) + 48000(8)$ $= R 624 000$	✓ substitution ✓ answer (2) [14]

TOTAL: 150