



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## **NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2**

**EXEMPLAR 2014**

**MEMORANDUM**

**MARKS: 150**

**This memorandum consists of 13 pages.**

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**QUESTION 1**

1.1	As the number of days that an athlete trained increased, the time taken to run the 100m event decreased. <b>OR</b> The fewer number of days an athlete trained, the longer the time he took to complete the 100m sprint. <b>OR</b> The greater number of days an athlete trained, the shorter the time he ran the 100m sprint.	✓ explanation          (1)
1.2	(60 ; 18,1)	✓   (1)
1.3	$a = 17,81931464...$ $b = -0,070685358...$ $\therefore \hat{y} = -0,07x + 17,82$	✓✓ $a$ ✓ $b$ ✓ equation  (4)
1.4	$\therefore \hat{y} \approx -0,07(45) + 17,82$ $\approx 14,67$ seconds	✓ substitution ✓ answer  (2)
1.5	$r = -0,74$ (-0,740772594...)	✓✓ $r$   (2)
1.6	There is a moderately strong relationship between the variables.	✓ moderately strong   (1) <b>[11]</b>

**QUESTION 2**

<p>2.1</p>		<ul style="list-style-type: none"> <li>✓ grounding at 0</li> <li>✓ plotting at upper limits</li> <li>✓ smooth shape of curve</li> </ul> <p style="text-align: right;">(3)</p>
<p>2.2</p>	<p><math>40 \leq t &lt; 60</math></p>	<ul style="list-style-type: none"> <li>✓ class</li> </ul> <p style="text-align: right;">(1)</p>
<p>2.3</p>	<p>(96 ; 164)  <math>\therefore 172 - 164 = 8</math> learners</p>	<ul style="list-style-type: none"> <li>✓ 164</li> <li>✓ 8</li> </ul> <p style="text-align: right;">(2)</p>
<p>2.4</p>	<p>Frequency: 25; 44; 60; 28; 9; 6  <math display="block">\text{Mean} = \frac{25 \times 10 + 44 \times 30 + 60 \times 50 + 28 \times 70 + 9 \times 90 + 6 \times 110}{172}</math> <math display="block">= \frac{8000}{172}</math> <math display="block">= 46,51 \text{ hours}</math></p>	<ul style="list-style-type: none"> <li>✓ frequency</li> <li>✓ midpoints</li> <li>✓ <math>\frac{8000}{172}</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(4)  <b>[10]</b></p>

**QUESTION 3**

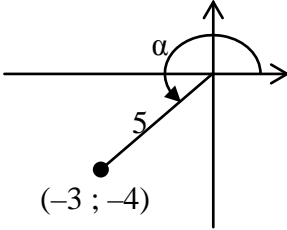
3.1	$K(7 ; 0)$	✓ answer (1)
3.2	$1 = \frac{x_M + 7}{2}$ and $1 = \frac{y_M + 3}{2}$ $\therefore M(-5 ; -1)$	✓ $x$ ✓ $y$ (2)
3.3	$m_{PM} = \frac{3-1}{7-1}$ $= \frac{1}{3}$	✓ substitution ✓ answer (2)
3.4	$\tan \hat{P}SK = m_{PM} = \frac{1}{3}$ $\hat{P}SK = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$ $\therefore \theta = 180^\circ - 90^\circ - 18,43^\circ = 71,57^\circ$	✓ $\tan \hat{P}SK = m_{PM}$ ✓ $\hat{P}SK$ ✓ $\theta$ (3)
3.5	$\cos 71,57^\circ = \frac{PK}{PS} = \frac{3}{PS}$ $PS = \frac{3}{\cos 71,57^\circ}$ $= 9,49$ units <b>OR</b> $\sin 18,43^\circ = \frac{PK}{PS} = \frac{3}{PS}$ $PS = \frac{3}{\sin 18,43^\circ}$ $= 9,49$ units	✓ correct ratio ✓ PS as subject ✓ answer (3) ✓ correct ratio ✓ PS as subject ✓ answer (3)
3.6	$N(x ; -2x + 17)$ $m_{TN} = m_{PM}$ (TN    PM) $\frac{-2x + 17 - 5}{x - (-1)} = \frac{1}{3}$ $-6x + 36 = x + 1$ $-7x = -35$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore N(5 ; 7)$ <b>OR</b>	✓ N in terms of $x$ ✓ equal gradients ✓ substitution ✓ $x$ -value ✓ $y$ -value (5)

	$m_{TM} = \frac{1}{3} \quad (\text{TN} \parallel \text{PM})$ <p>equation of TM:</p> $y - y_1 = \frac{1}{3}(x - x_1)$ $y - 5 = \frac{1}{3}(x - (-1))$ $y - 5 = \frac{1}{3}x + \frac{1}{3}$ $y = \frac{1}{3}x + 5\frac{1}{3}$ <p style="text-align: center;"><b>OR</b></p> $y = \frac{1}{3}x + c$ $5 = \frac{1}{3}(-1) + c$ $5\frac{1}{3} = c$ $y = \frac{1}{3}x + 5\frac{1}{3}$ $-2x + 17 = \frac{1}{3}x + 5\frac{1}{3}$ $-2\frac{1}{3}x = -11\frac{2}{3}$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore N(5 ; 7)$	<p>✓ <math>m_{TM}</math></p> <p>✓ equation of TM</p> <p>✓ equating</p> <p>✓ <math>x</math>-value</p> <p>✓ <math>y</math>-value</p> <p style="text-align: right;">(5)</p>
<p>3.7.1</p>	<p><math>y = 5</math></p>	<p>✓ equation</p> <p style="text-align: right;">(1)</p>
<p>3.7.2</p>	<p>gradient of AQ = <math>\tan 45^\circ</math> or <math>\tan 135^\circ</math>  <math>= 1</math> or <math>-1</math></p> $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore a - 1 = 4 \text{ or } -4$ $\therefore a = 5 \text{ or } -3$	<p>✓ <math>m_{AQ} = 1</math> or</p> <p>✓ <math>m_{AQ} = -1</math></p> <p>✓ substitution into gradient formula</p> <p>✓ <math>x</math>-value</p> <p>✓ <math>y</math>-value</p> <p style="text-align: right;">(5)  <b>[22]</b></p>

**QUESTION 4**

4.1	M(-1 ; -1)	✓ answer (1)
4.2	$m_{NT} = \frac{2-1}{3-4} = -1$ $\therefore m_{AT} = 1 \quad (\text{radius} \perp \text{tangent})$ $y - 1 = 1(x - 4)$ $y = x - 3$	✓ $m_{NT}$ ✓ $m_{AT}$ ✓ reason ✓ substitution of $m$ and (4 ; 1) ✓ equation (5)
4.3	$\text{MR} \perp \text{AB} \quad (\text{line from centre to midpt of chord})$ $\text{MB}^2 = \text{MR}^2 + \text{RB}^2 \quad (\text{Theorem of Pythagoras})$ $9 = \left(\frac{\sqrt{10}}{2}\right)^2 + \text{RB}^2$ $\text{RB}^2 = \frac{13}{2}$ $\text{RB} = \sqrt{\frac{13}{2}}$ $\text{AB} = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26} \text{ units}$	✓ $\text{MR} \perp \text{AB}$ ✓ $\text{MB} = 3$ ✓ substitution into Theorem of Pythagoras ✓ $\text{AB}$ in surd form (4)
4.4	$\text{MN}^2 = (-1 - 3)^2 + (-1 - 2)^2$ $= 16 + 9$ $= 25$ $\text{MN} = 5 \text{ units}$	✓ substitution into distance formula ✓ answer (2)
4.5	$r = 5 - 3 = 2 \text{ units}$ $\therefore (x - 3)^2 + (y - 2)^2 = 4$ $\therefore x^2 + y^2 - 6x - 4y + 9 = 0$	✓ $r$ ✓ substitution into circle equation ✓ equation (3) <b>[15]</b>

**QUESTION 5**

5.1.1	$-\sin \alpha$ $= -\left(-\frac{4}{5}\right) = \frac{4}{5}$	✓ reduction ✓ answer (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos \alpha = \frac{-3}{5}$ 	✓ $b = -3$ ✓ answer (2)
5.1.3	$\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{1}{\sqrt{2}} - \left(-\frac{3}{5}\right) \cdot \frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ <p style="text-align: center;"><b>OR</b></p> $\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{3}{5}\right) \cdot \frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ expansion ✓ $\frac{1}{\sqrt{2}}$ ✓ answer in simplest form (3)
5.2.1	$LHS = \frac{8 \sin x \cdot \cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2 \sin x \cdot \cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4 \sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4 \sin 2x}{-\cos 2x}$ $= -4 \tan 2x$	✓ $\sin x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ $4 \sin 2x$ ✓ factorise ✓ $-\cos 2x$ (6)
5.2.2	Undefined when $\cos 2x = 0$ or $\tan 2x = \infty$ : $x = 45^\circ$ and $x = 135^\circ$	✓ $45^\circ$ ✓ $135^\circ$ (2)

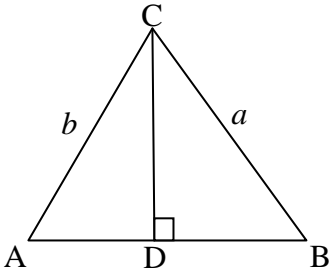
5.3	$1 - 2\sin^2 \theta + 4\sin^2 \theta - 5\sin \theta - 4 = 0$ $2\sin^2 \theta - 5\sin \theta - 3 = 0$ $(2\sin \theta + 1)(\sin \theta - 3) = 0$ $\therefore \sin \theta = -\frac{1}{2} \quad \text{or} \quad \sin \theta = 3 \text{ (no solution)}$ $\therefore \theta = 210^\circ + 360^\circ k \quad \text{or} \quad \theta = 330^\circ + 360^\circ k \quad ; k \in \mathbb{Z}$ <p><b>OR</b></p> $\therefore \theta = 210^\circ + 360^\circ k \quad \text{of} \quad \theta = 30^\circ + 360^\circ k \quad ; k \in \mathbb{Z}$	$\checkmark 1 - 2\sin^2 \theta$ $\checkmark$ standard form $\checkmark$ factors $\checkmark$ no solution $\checkmark 210^\circ$ $\checkmark 330^\circ$ $\checkmark + 360^\circ k \quad ; k \in \mathbb{Z}$ (7) <b>[22]</b>
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**QUESTION 6**

6.1	$b = \frac{1}{2}$	$\checkmark$ value of $b$ (1)
6.2	A(30° ; 1)	$\checkmark 30^\circ$ $\checkmark 1$ (2)
6.3	$x = 160^\circ$	$\checkmark x = 160^\circ$ (1)
6.4	$h(x) = 2\cos(x - 30^\circ) + 1$ $y \in [-1 ; 3]$ <b>OR</b> $-1 \leq y \leq 3$	$\checkmark$ critical values $\checkmark$ notation (2) <b>[6]</b>



**QUESTION 7**

<p>7.1</p>	<p>Draw <math>CD \perp AB</math>                  In <math>\triangle ACD</math>:  <math>\sin A = \frac{CD}{b} \therefore CD = b \cdot \sin A</math>                   In <math>\triangle CBD</math>:  <math>\sin B = \frac{CD}{a} \therefore CD = a \cdot \sin B</math>   <math>\therefore b \cdot \sin A = a \cdot \sin B</math>  <math>\therefore \frac{\sin A}{a} = \frac{\sin B}{b}</math></p> 	<p>✓ construction                  ✓ sin A                  ✓ making CD the subject                   ✓ sin B                   ✓ <math>b \cdot \sin A = a \cdot \sin B</math>                  (5)</p>
<p>7.2.1</p>	<p><math>\hat{S}PQ = 180^\circ - 2x</math> (opp <math>\angle</math>s of cyclic quad )  <math>\hat{P}SQ + \hat{P}QS = 2x</math> (sum of <math>\angle</math>s in <math>\triangle</math>)  <math>\hat{P}SQ = \hat{P}QS = x</math> (<math>\angle</math>s opp equal sides)</p>	<p>✓ <math>\hat{S}PQ = 180^\circ - 2x</math> (S/R)                  ✓ reason                  (2)</p>
<p>7.2.2</p>	$\frac{\sin \hat{S}PQ}{\sin(180^\circ - 2x)} = \frac{\sin \hat{P}SQ}{\sin x}$ $\frac{SQ}{\sin(180^\circ - 2x)} = \frac{PQ}{\sin x}$ $SQ = \frac{k \sin 2x}{\sin x}$ $SQ = \frac{k(2 \sin x \cdot \cos x)}{\sin x} = 2k \cos x$ <p style="text-align: center;"><b>OR</b></p> $SQ^2 = PQ^2 + PS^2 - 2PQ \cdot PS \cdot \cos \hat{S}PQ$ $= k^2 + k^2 - 2 \cdot k \cdot k \cdot \cos(180^\circ - 2x)$ $= 2k^2 + 2k^2 \cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2 \cos^2 x$ $SQ = 2k \cos x$	<p>✓ substitution into correct formula                  ✓ sin 2x                   ✓ SQ subject                  ✓ <math>2 \sin x \cdot \cos x</math>                  (4)                   ✓ substitution into correct formula                  ✓ <math>-\cos 2x</math>                  ✓ <math>2\cos^2 x - 1</math>                  ✓ simplification                  (4)</p>
<p>7.2.3</p>	$\tan y = \frac{3}{k}$ $k = \frac{3}{\tan y}$ $SQ = 2 \cos x \left( \frac{3}{\tan y} \right)$ $\therefore = \frac{6 \cos x}{\tan y}$	<p>✓ tan ratio                   ✓ k subject and substitution                   (2)  <b>[13]</b></p>

**QUESTION 8**

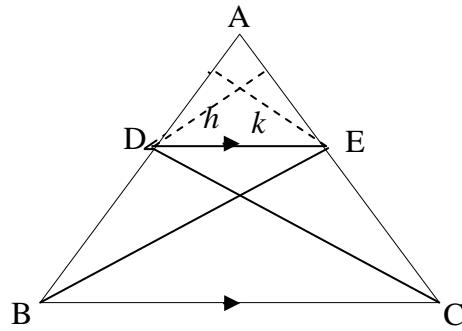
8.1	the angle subtended by the chord in the alternate segment	✓ correct theorem (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68^\circ$ (tan chord theorem)	✓ $\hat{E}_1 = 68^\circ$ ✓ reason (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68^\circ$ (alt $\angle$ s; $AE \parallel BC$ )	✓ $\hat{B}_3 = 68^\circ$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68^\circ$ (ext $\angle$ of cyclic quad)	✓ $\hat{D}_1 = 68^\circ$ ✓ reason (2)
8.2.4	$\hat{E}_2 = 20^\circ + 68^\circ$ $= 88^\circ$ (ext $\angle$ of $\Delta$ )	✓ $\hat{E}_2 = 88^\circ$ (S/R) (1)
8.2.5	$\hat{C} = 180^\circ - 88^\circ$ $= 92^\circ$ (opp $\angle$ s of cyclic quad)	✓ $\hat{C} = 92^\circ$ ✓ reason (2) <b>[9]</b>

**QUESTION 9**

<p>9.1</p>	<p><math>\hat{D}_4 = \hat{A} = x</math> (tan chord theorem)  <math>\hat{A} = \hat{D}_2 = x</math> (<math>\angle</math>s opp equal sides)</p>	<p>✓ <math>\hat{A} = x</math>                  ✓ reason                  ✓ <math>\hat{A} = \hat{D}_2 = x</math>                  (S/R)                  (3)</p>
<p>9.2</p>	<p><math>\hat{M}_1 = 2x</math> (ext <math>\angle</math> of <math>\Delta</math>) or (<math>\angle</math> at centre = <math>2\angle</math> at circum)  <math>\hat{M}\hat{D}E = 90^\circ</math> (radius <math>\perp</math> tan)  <math>\hat{M}_2 = 90^\circ - 2x</math>  <math>\therefore \hat{E} = 180^\circ - (90^\circ + 90^\circ - 2x)</math> (sum of <math>\angle</math>s in <math>\Delta MDE</math>)  <math>= 2x</math>  <math>\therefore CM</math> is a tangent (converse tan chord theorem)</p>	<p>✓ <math>\hat{M}_1 = 2x</math> (S/R)                  ✓ <math>\hat{M}\hat{D}E = 90^\circ</math>                  (S/R)                  ✓ <math>\hat{E} = 2x</math>                  ✓ reason                  (4)</p>
<p>9.3</p>	<p><math>\hat{M}_3 = 90^\circ</math> (<math>EM \perp AC</math>)  <math>\hat{A}\hat{D}B = 90^\circ</math> (<math>\angle</math> in semi-circle)  <math>\therefore FMBD</math> a cyclic quad (ext <math>\angle</math> of quad = int opp <math>\angle</math>)  <b>OR</b>  <math>\hat{E}\hat{M}C = 90^\circ</math> (<math>EM \perp AC</math>)  <math>\hat{A}\hat{D}B = 90^\circ</math> (<math>\angle</math> in semi-circle)  <math>\therefore FMBD</math> a cyclic quad (opp <math>\angle</math>s of quad supp)</p>	<p>✓ <math>\hat{M}_3 = 90^\circ</math>                  ✓ <math>\hat{A}\hat{D}B = 90^\circ</math> (S/R)                  ✓ reason (3)                  ✓ <math>\hat{E}\hat{M}C = 90^\circ</math>                  ✓ <math>\hat{A}\hat{D}B = 90^\circ</math> (S/R)                  ✓ reason (3)</p>
<p>9.4</p>	<p><math>DC^2 = MC^2 - MD^2</math> (Theorem of Pythagoras)  <math>= (3BC)^2 - (2BC)^2</math> (<math>MB = MD =</math> radii)  <math>= 9BC^2 - 4BC^2</math>  <math>= 5BC^2</math></p>	<p>✓ Th of Pythagoras                  ✓ substitution                  ✓ <math>9BC^2 - 4BC^2</math>                  (3)</p>
<p>9.5</p>	<p>In <math>\Delta DBC</math> and <math>\Delta DFM</math>:  <math>\hat{D}_4 = \hat{D}_2 = x</math> (proven in 9.1)  <math>\hat{B}_1 = \hat{F}_2</math> (ext <math>\angle</math> of cyclic quad)  <math>\hat{C} = \hat{M}_2</math>  <math>\therefore \Delta DBC \parallel \parallel \Delta DFM</math> (<math>\angle</math>; <math>\angle</math>; <math>\angle</math>)</p>	<p>✓ <math>\hat{D}_4 = \hat{D}_2</math>                  ✓ <math>\hat{B}_1 = \hat{F}_2</math>                  ✓ reason                  ✓ <math>\hat{C} = \hat{M}_2</math> or                  (<math>\angle</math>; <math>\angle</math>; <math>\angle</math>)                  (4)</p>
<p>9.6</p>	<p><math>\frac{DM}{FM} = \frac{DC}{BC}</math> (<math>\Delta DBC \parallel \parallel \Delta DFM</math>)  <math>= \frac{\sqrt{5}BC}{BC}</math>  <math>= \sqrt{5}</math></p>	<p>✓ S                  ✓ answer                  (2)  <b>[19]</b></p>

**QUESTION 10**

10.1



Construction: Join DC and BE and heights  $k$  and  $h$

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\frac{1}{2} \cdot AD \cdot k}{\frac{1}{2} \cdot DB \cdot k} = \frac{AD}{DB} \quad (\text{equal heights})$$

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2} \cdot AE \cdot h}{\frac{1}{2} \cdot EC \cdot h} = \frac{AE}{EC} \quad (\text{equal heights})$$

But Area  $\triangle DEB = \text{Area } \triangle DEC$  (same base, same height)

$$\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

✓ construction

$$\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{AD}{DB}$$

✓ reason

$$\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$$

✓ Area  $\triangle DEB = \text{Area } \triangle DEC$  (S/R)

✓

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$$

(6)

<p>10.2.1</p>	$\frac{AB}{BE} = \frac{AC}{CD}$ <p>(Prop Th; BC    ED)</p> $\frac{1}{3} = \frac{3}{CD}$ $\therefore CD = 9 \text{ units}$	<p>✓ <math>\frac{AB}{BE} = \frac{AC}{CD}</math> (S/R)</p> <p>✓ substitution</p> <p>✓ answer</p> <p>(3)</p>
<p>10.2.2</p>	$\frac{DG}{GA} = \frac{FD}{FE}$ <p>(Prop Th; FG    EA)</p> $\frac{9-x}{3+x} = \frac{3}{6}$ $54 - 6x = 9 + 3x$ $-9x = -45$ $x = 5$	<p>✓ <math>\frac{DG}{GA} = \frac{FD}{FE}</math> (S/R)</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ answer</p> <p>(4)</p>
<p>10.2.3</p>	<p>In <math>\triangle ABC</math> and <math>\triangle AED</math>:</p> <p><math>\hat{A}</math> is common</p> <p><math>\hat{ABC} = \hat{E}</math> (corres <math>\angle</math>s; BC    ED)</p> <p><math>\hat{ACB} = \hat{D}</math> (corres <math>\angle</math>s; BC    ED)</p> <p><math>\triangle ABC \sim \triangle AED</math> (<math>\angle, \angle, \angle</math>)</p> $\therefore \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{9} = \frac{3}{12}$ $BC = 2\frac{1}{4} \text{ units}$	<p>✓ <math>\hat{A}</math> is common</p> <p>✓ <math>\hat{ABC} = \hat{E}</math> (S/R)</p> <p>✓ <math>\hat{ACB} = \hat{D}</math> (S/R)</p> <p>or (<math>\angle; \angle; \angle</math>)</p> <p>✓ <math>\frac{BC}{ED} = \frac{AC}{AD}</math></p> <p>✓ answer</p> <p>(5)</p>
<p>10.2.4</p>	$\frac{\text{area } \triangle ABC}{\text{area } \triangle GFD} = \frac{\frac{1}{2} AC \cdot BC \cdot \sin \hat{ACB}}{\frac{1}{2} GD \cdot FD \cdot \sin \hat{D}}$ $= \frac{\frac{1}{2} (3)(2\frac{1}{4}) \sin \hat{D}}{\frac{1}{2} (4)(3) \sin \hat{D}}$ <p>(corres <math>\angle</math>s; BC    ED)</p> $= \frac{9}{16}$	<p>✓ use of area rule</p> <p>✓ correct sides and angles</p> <p>✓ substitution of values</p> <p>✓ <math>\sin \hat{ACB} = \sin \hat{D}</math> (S/R)</p> <p>✓ answer</p> <p>(5)</p> <p>[23]</p>

**TOTAL: 150**